Apeiron/AME-1 — Canonical Operational Model Specification

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1. Scope and epistemic boundary

This is the canonical operational definition of the homogeneous model that produced the frozen v7.13 classification. It specifies variables, pressure function, background equations, quantum-source closure, discrete fixed-point map, initial data, units and conventions.

It does **not** assert a unique fundamental covariant action, perturbation theory, ontology or empirical confirmation. For the narrowly scoped numerical-foundations paper, the equations and executable map below are authoritative. An action-level completion would be new physics and cannot be projected backward onto v7.13.

2. Units, geometry and time

- Natural units: $c = \hbar = 1$.
- Reduced Planck mass as internal mass unit: $M_{pl} = 1$.
- Spatially flat homogeneous FRW background, scale factor a, e-fold variable $N = \ln(a)$, expanding branch $H > 0$.
- A dot denotes differentiation with respect to cosmic time t.
- Dimensionless numerical time:

  $\tau = H_{star}\, t$
  $d/d\tau = (1/H_{star})\, d/dt$

- Noncanonical kinetic scalar:

  $X = 0.5\, (\dot\theta)^2$

- Scalar fields and mass parameters have mass dimension one. X, pressure and energy density have mass dimension four.

The operational sign convention is fixed by the equations below. A metric-signature convention not used explicitly by the homogeneous implementation is not added here.

3. Degrees of freedom and stored state

3.1 Homogeneous state

The full eight-column trajectory container is

`y(τ) = (σ, dot σ, θ, dot θ, χ, dot χ, H, N).`

In the registered v7.13 map the homogeneous bookkeeping coordinate χ is fixed to zero. The dynamically iterated six-column state is

`u(τ) = (σ, dot σ, θ, dot θ, H, N).`

The χ sector enters through the renormalized mode expectation values $\rho_q(\tau)$, $p_q(\tau)$ and $\_q(\tau)$, not through an independently iterated homogeneous χ.

3.2 Discrete vector

The frozen grid has N_τ = 15361 nodes. The solver vector is the flattened and scaled array

```
x = vec[
  σ/(1e-2),
  dot σ/(1e-8),
  θ/1,
  dot θ/(1e-8),
  H/(1e-6),
  N/(1e-1)
]
```

with shape (92166,) and Float64 dtype.

Array	Shape	dtype	Meaning
t	(15361,)	Float64	dimensionless τ grid
y	(15361, 8)	Float64	full trajectory container
x	(92166,)	Float64	scaled flattened six-column iterate
tau	scalar	Float64	active terminal value

During unpacking, first-node σ, dot σ, θ, dot θ and N are reset to the frozen background initial data. First-node H remains in the active state; the image map then imposes the positive Friedmann closure.

3.3 Auxiliary and constrained quantities

- H is algebraically reconstructed from the positive Friedmann branch inside the registered image map.
- N obeys dot N = H.
- Quantum modes and Pauli–Villars regulator sectors are auxiliary arrays used to construct ρ_q, p_q and _q; they are not independent entries of x.
- Gate diagnostics are not additional dynamical degrees of freedom.

4. Frozen parameters

4.1 AME parameters

Symbol	Value	Code name
M_pl	1	Mpl
Λ	2.3e-3	Lambda
f_σ	2.5e-2	f_sigma
E_c / V_star	4.0e-2	Ec_over_V
H_star	2.0e-6	Hstar
b_star	0.35	bstar
d b / d(θ/M_pl)	-4.78e-2	b_slope
A_eff	6.86e-3	Aeff
B_eff	-1.74e-2	Beff

```
V_star = 3 M_pl² H_star²
E_c = 0.04 V_star
```

A_eff and B_eff remain in parameter provenance; the active global pressure completion in Section 5 is the quartic polynomial V_s(θ).

4.2 Portal and regulator parameters

Quantity	Value
phase scale f_phase	0.8549468502440697
displacement v	0.03272387838931097
coupling λ	1.6740193604000272e-5
bare portal mass m_χ,0	9.817951142008693e-5
physical-mode support at initial slice	0 ≤ k ≤ 0.6 Λ
physical quadrature nodes	128
renormalization scale	μ_R = 1.5 Λ
UV order-0 switch	16 Λ
curvature-tail limit	64 Λ
tail nodes per octave	20

5. Effective AME pressure and derivatives

Define

```
z = σ/f_σ
b(X, θ) = b_star + b_slope (θ/M_pl) - 5X/Λ⁴
F(z,b) = z⁴/4 - z²/2 + bz
V_s(θ) = Σ(i=0..4) c_i θ^i
```

with

```
(c_0,c_1,c_2,c_3,c_4) =
( 1.1971212004620583e-11,
  9.87022611239063e-14,
 -1.0336424891583054e-13,
  1.189490499106138e-10,
  1.2236365189926054e-10 )
```

The homogeneous pressure function is

```
P(X,σ,θ) =
  X
  + [5/(3Λ⁴)] (z - 2/√3)² X²
  - E_c F(z,b)
  - V_s(θ).
```

The executable derivatives are

```
P_X =
  1
  + [10/(3Λ⁴)] (z - 2/√3)² X
  + 5 E_c z/Λ⁴

P_XX = [10/(3Λ⁴)] (z - 2/√3)²

P_σ =
  { [10/(3Λ⁴)] (z - 2/√3) X²
    - E_c (z³ - z + b) } / f_σ

P_θ = -E_c z b_slope/M_pl - V_s'(θ)

P_Xσ =
  { [20/(3Λ⁴)] (z - 2/√3) X
    + 5 E_c/Λ⁴ } / f_σ.
```

AME stress:

```
ρ_AME = 0.5 (dot σ)² + 2X P_X - P
p_AME = 0.5 (dot σ)² + P.
```

Hyperbolicity diagnostics:

```
P_X > 0
P_X + 2X P_XX > 0.
```

6. Portal and quantum source

Field-dependent portal mass:

```
m_χ²(σ,θ) =
  m_χ,0² + 2λ(v+σ)² cos(2θ/f_phase)

∂m_χ²/∂σ = 4λ(v+σ) cos(2θ/f_phase)

∂m_χ²/∂θ =
  -[4λ(v+σ)²/f_phase] sin(2θ/f_phase).
```

Four Pauli–Villars sectors:

```
C_j = (1,-3,3,-1)
J_j = (0,1,2,3)
M_j² = m_χ² + J_j μ_R².
```

Modes are advanced by a three-substep Yoshida composition of the implicit-midpoint map for $u = a^{(3/2)} \chi$. Initial mode data are

```
u_k(0) = 1/sqrt(2ω_k(0))
dot u_k(0) = -i ω_k(0) u_k(0).
```

The active source combines resolved modes, the analytic order-0 UV tail and auxiliary adiabatic curvature orders 2 and 4. Finite renormalization is fixed on the initial flat-space reference slice as recorded by the active runtime:

- $\rho_q(0) = 0$;
- _q(0) = 0;
- initial quantum pressure is predicted by the fixed prescription rather than tuned as a gate;
- the Einstein–Hilbert response is fixed by the zero-curvature response;
- the finite R^2 coefficient is zero.

The routine returns $\rho_q(\tau)$, $p_q(\tau)$ and _q(τ). Its source identity is

```
dot ρ_q + 3H(ρ_q+p_q)
  - 0.5 (dot m_χ²) <χ²>_q = 0,
```

evaluated as a normalized numerical diagnostic.

7. Registered homogeneous equations

With

```
ρ_tot = ρ_AME + ρ_q
p_tot = p_AME + p_q,
```

the registered image map integrates

```
dσ/dt = v_σ

dv_σ/dt =
```

$$
\begin{aligned}
&-3H\, v_\sigma + P_\sigma \\
&- 0.5\, (\partial m_\chi^2/\partial\sigma)\, \langle\chi^2\rangle_q
\end{aligned}
$$

$$d\theta/dt = v_\theta$$

$$
\begin{aligned}
dv_\theta/dt = {} & \{ P_\theta \\
& \quad - 0.5\, (\partial m_\chi^2/\partial\theta)\, \langle\chi^2\rangle_q \\
& \quad - 3H\, P_X\, v_\theta \\
& \quad - P_{X\sigma}\, v_\sigma\, v_\theta \} \\
& / (P_X + 2X\, P_{XX})
\end{aligned}
$$

$$H = +\mathrm{sqrt}[\rho_{tot}/(3\, M_{pl}^2)]$$

$$dN/dt = H.$$

The Raychaudhuri relation

$$dH/dt = -(\rho_{tot}+p_{tot})/(2\, M_{pl}^2)$$

is evaluated independently rather than used as the evolution equation in this image map.

8. Initial data

$$
\begin{aligned}
&\theta(0) = -0.690 \\
&\varepsilon_0 = 2.35\mathrm{e}{-5} \\
&N(0) = 0
\end{aligned}
$$

$$
\begin{aligned}
\dot\theta(0) = {} & \\
& -\mathrm{sqrt}(2\, \varepsilon_0)\, M_{pl}\, H_{star} / \mathrm{sqrt}(1.3)
\end{aligned}
$$

$$\dot\sigma(0) = 0.$$

$\sigma(0) = f_\sigma\, z_+$, where z_+ is the positive local minimum satisfying

$$
\begin{aligned}
&z^3 - z + b(\theta(0)) = 0 \\
&3z^2 - 1 > 0.
\end{aligned}
$$

The initial Hubble value is the positive Friedmann root. Homogeneous portal bookkeeping data are $\chi(0) = \dot\chi(0) = 0$. No spatial boundary conditions occur because the registered model is homogeneous.

9. Canonical discrete fixed-point map

For active endpoint τ_{end},

$$\Delta\tau = \tau_{end} / 15360.$$

The canonical image map $G(x;\tau_{end})$:

1. creates the uniform 15361-node τ grid and frozen background seed;
2. unpacks x into the six active trajectory columns using Section 3.2;
3. imposes frozen first-node conditions;
4. calculates renormalized quantum-source arrays with the fixed portal and PV prescription;
5. advances (σ, $\dot\sigma$, θ, $\dot\theta$, N) by classical fourth-order Runge–Kutta while imposing the positive Friedmann root for H;
6. lifts the result to the eight-column container and repacks the six active columns.

Residual and fixed-point diagnostic:

```
R(x;τ) = x - G(x;τ)
δ_FP = ||R||_infinity.
```

This is complete for the frozen homogeneous discrete map. It does not license continuum, perturbative, global-stability or observational claims.

10. Active-source provenance

Source file	SHA-256	Role
model.py	db4c0fa9cf4f2013fef299f558ca7b[illegible]ece6bd2dd0c5a65	parameters and pressure primitives
frw_noncanonical_background_v6_6.py	f96b13df83e6d6dce8f9bec073e2a[illegible]a327f2b61f7f1	homogeneous equations and initial data
frw_pv_fixed_mpl_renorm_v6_17.py	aecd4dc3202604228d1a46c1a4c[illegible]fbb5b999df48[illegible]	stress tensor, renormalization and diagnostics
frw_pv_covariant_uv_complete_v6_24.py	9c21b4095845687b9824dab66da[illegible]a5decd01055ef487b14b4c19[illegible]	UV-tail closure
frw_pv_uv_yoshida_v6_31.py	8fa1b93a2f24377c751f3d9fdffe7[illegible]35d19ae3943e1	registered mode evolution
rb19_anderson_uv_v6_24.py	8323bbf6c6f3e23e1d466addf5cc[illegible]34fddd50ce8fcef4ae04fcc2c	state packing
rb23_friedmann_closure_uv_v6_28.py	873d3eca436d28b426162395791[illegible]5509b9fb97d1fe4f5d7b	Friedmann map and Raychaudhuri audit
rb_recovery_fixedtau_v6_64.py	5082e44cf9c60ca9ef1eaa0ba1cc[illegible]c7581be82ee72	fixed-point and solver base
rb_recovery_adaptive_v7_13.py	8854c5847f9da3e67994701a20e[illegible]0fd7017f81205	[illegible] wrapper and schedule

Filename version labels record dependency origin. They appear only because these files form the active hash-frozen execution path of the Latest runtime. No alternative historical model is reconstructed.

11. Licensed publication statement

Maximum supported wording:

> Apeiron/AME-1 v7.13 is a frozen homogeneous FRW numerical model defined operationally by the equations, parameters, state map, quantum closure, discretization and hash-identified active runtime sources in this supplement.

Inadmissible stronger wording:

> The supplement proves a unique fundamental or covariant theory of Apeiron.

Current interpretive overlay - 14 September 2026

This section is an **interpretive addendum only**. It changes no frozen AME-1 equation, numerical parameter, gate, source hash or terminal classification.

Single-carrier principle

Apeiron is treated as one fundamental carrier. Effective sectors used in the operational model must not be read as ontologically separate fundamental substances.

- ordinary matter: organized/emergent Apeiron manifestation;
- dark-matter-like phenomena: possible Apeiron manifestation/state;
- dark-energy-like large-scale behavior: possible Apeiron dynamical/state manifestation;

- Higgs/QCD structures: possible organized low-energy manifestations under separate investigation, with Standard-Model/QCD structure retained;
- black-hole reconversion: possible transition of an organized manifestation toward more fundamental Apeiron states under extreme curvature.

The overlay is a model hypothesis and is not an empirical consequence of the frozen v7.13 calculation.

Thermodynamics, information and reconversion

The following is a binding current-facing model assumption and does not alter any frozen numerical gate or previously closed physical result.

Within the Apeiron framework, black holes and conversion regions are not interpreted as sinks in which information, energy, or physically relevant entropy disappear from the total system.

The sequence

access region → conversion region → directed phase return to Apeiron

is modeled as a transition between different physical organizations of the same underlying Apeiron system.

Observable matter degrees of freedom entering the conversion region are therefore assumed to be transformed into, or re-encoded within, Apeiron degrees of freedom. Energy and information are not postulated to be destroyed in this process.

Information conservation and entropy conservation are not identified with one another. The framework does not require the total entropy of the complete Apeiron system to remain constant. Entropy may be produced, redistributed, or transferred into Apeiron degrees of freedom during reconversion. The essential assumption is that physically relevant entropy is not simply erased by the phase transition.

The same distinction applies to Genesis. A newly formed observable matter phase may begin in a comparatively low-entropy state without requiring a decrease, erasure, or reset of the entropy of the complete Apeiron system. Entropy may remain stored, distributed, or reorganized within the larger Apeiron sector while the emergent observable phase occupies a lower-entropy configuration.

Accordingly, the Apeiron framework treats reconversion and Genesis as transformations within one larger physical system rather than as processes requiring the destruction or spontaneous resetting of information or entropy.

These statements are assumptions of the Apeiron framework. They are not presently claimed as experimentally established properties of black holes, quantum gravity, or cosmological phase transitions.

Remaining thermodynamic gap. A quantitative entropy balance, including a generalized-second-law-like consistency test across observable and Apeiron degrees of freedom, has not yet been produced and remains open.

14. Internal EFT / microscopic-completion supplement - Version 1.4

This supplement is additive and does not alter the frozen v7.13 operational map above.

14.1 Result class

- **Derived result:** controlled internal EFT vacuum and physically normalized σ, θ, and Z_2^χ-protected χ eigenmodes.
- **Derived result:** neighboring tachyonic χ phases are delimited and retained as adjacent phases.

- **Derived result:** perturbative/tree-level control reaches the nominal EFT scale.
- **Conditional UV interpretation:** the minimal structurally preferred matching class is a heavy linearly coupled Gaussian scalar mediator.
- **Bounded parameter:** the surviving minimal UV family has one free microscopic parameter M_S.
- **Unresolved question:** the physical nonlocal response kernel has not been produced, so M_S is not independently pole-fixed.

14.2 Binding corrected local coefficients

$$c_{X^2}^{(\phi_1)} = 3.91305380277 \times 10^{11} M_{\mathrm{Pl}}^{-4}.$$

$$\Lambda_{\mathrm{unit}} = 1.28824506\,\Lambda.$$

$$C_6^{(\chi)} = -21.0925 M_{\mathrm{Pl}}^{-6}.$$

$$\beta_{C_6} = -2.85893108646 \times 10^{12} M_{\mathrm{Pl}}^{-6}.$$

The old intermediate value 2.9131×10^{11} for the canonical X^2 coefficient is rejected. The old dependent ratio $\Lambda_{\mathrm{unit}}/\Lambda = 1.38688$ is likewise rejected. No closed physics is recomputed by this documentation update; the Version 1.4 release merely propagates the already closed correction.

14.3 UV boundary

APEIRON UV MATCHING BOUNDARY: BOUNDED

The minimal branch is a **quantitative one-parameter UV family** with

$$1 < M_S/\Lambda \lesssim 1.28824506.$$

This must not be rewritten as a unique microscopic completion.

14.4 Open nonlocal test

physical nonlocal response kernel still required

The absent physical nonlocal response kernel is the remaining route to a possible independent pole/mediator determination. Until that object is produced and validated, M_S remains bounded but not independently fixed.

15. Emergent causal structure supplement - Version 1.5

15.1 Status

Separate causality branch status: `FEASIBILITY_GO`. This supplement does not modify any frozen v7.13 equation, gate, source hash or numerical classification.

15.2 Canonical interpretation

Apeiron is not modeled here as a classical optical medium that slows photons from a higher ordinary speed. The allowed interpretation is that deeper Apeiron dynamics may generate a stable observable universe phase with one effective geometry and one universal finite causal cone. Observable light, matter/fermion and tensor/gravitational propagation then share that geometry at the effective level.

Minimal relation:

$$A \to G_{\mu\nu}[A] \to \{\gamma, \text{matter}, \text{gravity}\}, \qquad c_\gamma = c_{e,\text{limit}} = c_g = c.$$

A deeper monotonic Apeiron development parameter may exist, but it is not ordinary hidden photon proper time. A separate universal fundamental c_A is optional and not required.

15.3 Micro-EFT cone result

At the previously tested static Frozen vacuum, $X_0 = 0$ in the θ $P(X)$ sector and therefore

$$P_X(X_0) = 1, \qquad c_\theta^2 = \frac{P_X}{P_X + 2XP_{XX}} = 1.$$

With canonical kinetic terms for σ and χ on the same background,

$$c_\sigma^2 = c_\chi^2 = 1, \qquad c_\sigma = c_\theta = c_\chi.$$

Classification: `MICRO_CONE_PASS_AT_TREE_LEVEL`.

15.4 Open boundary

The common tree-level cone is a tested property of the existing micro-EFT around the specified background. It is not a derivation of the Lorentz structure from deeper pre-EFT Apeiron dynamics. The required fundamental closure is therefore: `FUNDAMENTAL_LORENTZ_ORIGIN_OPEN`. A first-principles derivation of universal mono-metricity also remains open.