What can one measured curve reveal?

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Identifiability and calibration limits of a single dispersion branch in a two-field effective theory

The question

When two oscillations are coupled, they form two collective modes. Often, only the lower-energy mode is measured. P01 asks whether these observations can also determine the second, higher-energy mode.

The result: Within a precisely specified model, this reconstruction is generally unique. Its physical accuracy, however, depends on calibration and justified error bounds.

What the mathematics establishes

Three exact observations generally suffice. At three distinct, known positive momentum magnitudes, lower-branch frequencies determine the stable two-field model's three spectral parameters. A separately known initial slope is not required. Two observations are generally insufficient.

An exact exception remains. At a = r, the lower curve is exactly linear in the squared variables. The upper energy scale is then undetermined even with arbitrarily many lower-frequency observations. An additional upper-gap measurement or an appropriately calibrated response residue can provide the missing information.

Uniqueness alone does not guarantee accuracy. For known momenta, the robustness analysis translates justified measurement and model errors into finite parameter bounds. A separate extension includes momentum errors. Both require jointly valid error bounds and a justified domain of validity.

What the model extension adds

An explicitly selected additional classical field allows model corrections to be derived from the action. This Gaussian matching provides a controllable example. It is neither a complete quantum calculation nor evidence that the Foundation physically realizes this extension. The reference model's upper scale must be distinguished from the actual upper mode of an extended theory.

A test using published measurements

What was examined

The data of Mueller and colleagues describe gold nanoparticle crystals. The archive contains 167 previously extracted lower-dispersion points for five particle sizes. Layer numbers are explicitly assigned for the 41 nm sample; the original spectra were also audited. [1, 2]

This application concerns the separately derived local boundary case a = 0. It does not validate the original strict a > 0 case. No independently measured upper mode is available for this test.

Comparing the same five held-out points

For each layer group, two outer published points determine the fitted parameters. Predictions are then compared on the same five interior points from groups with four to seven layers.



RMSE is the root mean square prediction error. Both descriptions have two fitted parameters per group; Hopfield additionally uses a fixed nominal momentum calibration. This retrospective evaluation of known points is not an independent confirmation or a statistical significance test. The RMSE advantage is only 4.90 meV. [3]

How much tolerance does the model require?

With nominal calibration fixed, all points in a group, including the outer points, may now move within the same energy tolerance. The smallest tolerance admitting any physically admissible model curve was checked using exact enclosures:

Layers	4	5	6	7
Minimum tolerance	17.10 meV	11.89 meV	0.0194 meV	4.58 meV

These are mathematical compatibility thresholds, not measured error bars. In particular, the small six-layer value does not establish experimental precision at that level. [3]

What is supported, and what remains open?

A good lower curve does not certify the upper energy

Converting the tabulated momentum requires a common scale determined by geometry and refractive index. If two lower points are held exact while this scale is completely unconstrained, the calculated upper energy can grow without bound as the lower curve approaches a finite saturation curve. Large scale changes are mathematical comparison cases, not demonstrated alternatives to the actual geometry.

Three compatible exact points can, in principle, identify the additional scale. With finite errors, the question is whether the data and external calibration constrain it tightly enough. [3]

Looking inside the original spectra

All 15 archived absorbance, reflectance and transmittance files were verified against repository checksums. The 41 nm spectra cover two to seven layers. Absorbance is calculated from reflectance and transmittance; these are not three independent repeats.

The lowest published six-layer peak is at 0.78068 eV, while its archived spectrum starts at 0.8 eV. The reason for this difference is undocumented. Complete repeat data, joint error bounds and the full original fitting recipe are also unavailable. A controlled implementation of several fitting variants does not resolve the mode-assignment and boundary problems or provide reliable peak-center errors. [2, 3]

The defensible conclusion

P01 provides a conditional reconstruction and testing framework, concrete data checks and explicit limits. A physical upper-mode certificate still requires jointly valid, independently justified bounds on measurement and model discrepancies, including the common momentum calibration, and a verifiable spectral assignment. The Foundation theory is not validated by this work.

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Sources and numerical basis

[1] Mueller et al., Nature 583, 780-784 (2020). DOI: 10.1038/s41586-020-2508-1.

[2] Original data: DOI 10.17169/refubium-26819; CC BY-NC-SA 4.0. Spectra and published point assignments were audited separately.

[3] P01 main paper and Supplements S1-S4, publication edition 1.0-publication.1. The underlying research packages dated 23 September 2026 are retained in the internal research archive. This brief does not replace the main manuscript or technical supplements.