Apeiron: a source-bound local scalar effective theory

Definition, admissible domain and conditional consequences

Local theory draft, version 1.0 — 23 September 2026

Abstract

We consolidate the existing Apeiron scalar proposal into one explicitly defined local classical effective theory. The selected action retains the original noncanonical pressure, global quartic phase potential and screened quadratic portal, while distinguishing these choices from non-equivalent variants in the source record. The pressure's derivative-dependent tilt contributes to the quadratic phase normalization; retaining it corrects an earlier statement without changing the unit characteristic speed at a zero-gradient vacuum. We state the field equations, stress tensor, parameter conventions and initial-value requirements. In the admissible timelike-gradient domain, the phase characteristic speed is positive and no greater than the metric speed. The same action satisfies the null energy condition there, excluding a spatially flat nonsingular bounce within minimal Einstein gravity and that domain. An existing checked local matter spectrum supplies a benchmark, not an empirical observation. We formulate a conditional characteristic test and state the additional inputs needed for physical inference. The result is a delimited theory specification and its consequences; fundamental microstructure, nonlinear reheating, quantum completion and empirical confirmation remain open.

1 Purpose and relation to the Apeiron hypothesis

The Apeiron proposal combines a scalar-field description with broader hypotheses about the origin of matter, cosmological evolution and causal structure [1]. A useful theoretical core must specify which of those statements follow from an action, which are assumptions, and which require additional physics. We therefore formulate a single, explicitly delimited version of the existing model and state its equations and local domain before discussing consequences.

The construction follows the original pressure function and the global potential retained in the source record. It does not replace them by a separate illustrative two-field model. Recovered internal derivations help identify the intended covariant scalar sector, while the preprint provides the historical claim context. Neither a historical calculation nor the presence of a formula in an earlier document is treated as independent validation. The accompanying claim/source matrix records the precise evidence used for each statement and the corrections adopted in this paper.

Here, a *postulate* specifies the model, a *derived statement* follows within its stated assumptions, and a *conjecture* extends beyond those results. A locally admissible effective theory is a meaningful object for further analysis even when its parameters are not fixed by observations. It does not by itself establish a fundamental ontology, a unique ultraviolet completion or empirical adequacy. The results below must be read with that distinction throughout.

The pressure belongs to the established k-essence class; field-dependent terms linear and quadratic in X have long been studied [2]. Our claim concerns the explicit formulation and limits of this particular source model, not priority for that general mechanism.

2 One specified model

2.1 Source identity and conventions

The source pairing is preprint v1.6 (21 September 2026), its retained canonical specification v1.5 (20 September 2026), and the recovered S06 internal matter terms. The specification retains an older opening version heading; its file version and section identify the source. The selected matter action is reconstructed from the explicit covariant scalar terms in the recovered original internal sources and the frozen global functions used by their spectrum and cosmological runtime. Identifying that covariant matter theory with the operational homogeneous Apeiron proposal is a declared, source-based model postulate; the homogeneous preprint does not uniquely derive a covariant completion. We adopt the global quartic potential and screened portal benchmark throughout. The companion source matrix identifies the original messages, runtime definitions, preprint versions and existing independent checks [1, 3, 4]. General multi-field perturbation theory also shows why a homogeneous specification should not be assumed to fix every perturbative property [5].

We work in four dimensions with metric signature $(-,+,+,+)$ and units $\hbar = c = 1$. The reduced Planck mass is $M_{\rm Pl}$. The real fields σ, θ, χ have mass dimension one. Define

$$z = \frac{\sigma}{f_\sigma}, \qquad z_c = \frac{2}{\sqrt{3}}, \qquad X = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \theta \nabla_\nu \theta. \tag{1}$$

Thus X has mass dimension four. The coordinate θ is a real local phase coordinate, not a globally identified periodic field: the full potential contains a linear tilt and a polynomial, despite the periodic portal. The suggested parametrization $(v+\sigma)e^{i\theta/f_{\rm phase}}$ does not derive this action from a canonical complex scalar.

Use $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ and $T_{\mu\nu} = -2(-g)^{-1/2}\delta S_{\rm m}/\delta g^{\mu\nu}$. On a homogeneous spatially flat Friedmann–Robertson–Walker (FRW) background, $\Box\theta = -\ddot{\theta} - 3H\dot{\theta}$. Variations have compact support, or compatible boundary data and the required gravitational boundary terms are understood.

2.2 Action and exact coefficient functions

The matter postulate is

$$S_{\rm m} = \int {\rm d}^4x\sqrt{-g}\,\mathcal{L}_{\rm m}, \qquad \mathcal{L}_{\rm m} = -\frac{1}{2}(\nabla\sigma)^2 + P(X,\sigma,\theta) - \frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}m_\chi^2(\sigma,\theta)\chi^2, \tag{2}$$

with

$$F(z;b) = \frac{1}{4}z^4 - \frac{1}{2}z^2 + bz, \tag{3}$$

$$b_\theta(\theta) = b_* + b_1\frac{\theta}{M_{\rm Pl}}, \tag{4}$$

$$b(X,\theta) = b_\theta(\theta) - \frac{5X}{\Lambda^4}, \tag{5}$$

$$m_\chi^2 = m_{\chi 0}^2 + 2\lambda(v+\sigma)^2\cos\left(\frac{2\theta}{f_{\rm phase}}\right), \tag{6}$$

$$P = X + \frac{5(z-z_c)^2}{3\Lambda^4}X^2 - E_c F(z;b(X,\theta)) - V_s(\theta). \tag{7}$$

The exact equivalent form is

$$P = K(z)X + a(z)X^2 - U(\sigma, \theta), \tag{8}$$

$$K(z) = 1 + \frac{5E_c z}{\Lambda^4}, \qquad a(z) = \frac{5(z - z_c)^2}{3\Lambda^4}, \tag{9}$$

$$U = E_c \left[\frac{1}{4}z^4 - \frac{1}{2}z^2 + b_\theta(\theta)z\right] + V_s(\theta). \tag{10}$$

The contribution $5E_c z/\Lambda^4$ in K follows from the original X-dependent tilt. Omitting it changes the theory and its canonical normalization. The symbol $K(z)$ denotes a coefficient function, not the full kinetic matrix on every background. The function $a(z)$ is unrelated to an FRW scale factor.

The portal is quadratic in χ and has exact $\chi \mapsto -\chi$ symmetry. This finite truncation specifies neither a nonlinear saturation mechanism in a tachyonic region nor a quantum-complete portal theory.

2.3 Parameters and the retained benchmark

The dimension-one parameters are $\Lambda, f_\sigma, f_{\rm phase}, v, m_{\chi 0}$ and the reference normalization H_*. The parameters E_c, V_s have dimension four, a dimension minus four, and K, b_*, b_1, λ are dimensionless. Apart from the prescribed functional forms, parameters are model inputs. The following retained numerical point is an archival benchmark, not a determination from measured uncertainties.

Quantity	Selected value
$\Lambda/M_{\rm Pl}$	0.0023
$f_\sigma/M_{\rm Pl}$	0.025
$H_*/M_{\rm Pl}$	2×10^{-6}
$E_c/M_{\rm Pl}^4$	$4.8 \times 10^{-13} = 0.04 \times 3(H_*/M_{\rm Pl})^2$
$b_*,\ b_1$	$0.35,\ -0.0478$
$f_{\rm phase}/M_{\rm Pl}$	0.8549468502440697
$v/M_{\rm Pl}$	0.03272387838931097
λ	$1.6740193604000272 \times 10^{-5}$
$m_{\chi 0}/M_{\rm Pl}$	$9.817951142008693 \times 10^{-5}$

Write $\vartheta = \theta/M_{\rm Pl}$ and

$$V_s(\theta) = M_{\rm Pl}^4 \sum_{j=0}^{4} d_j \vartheta^j. \tag{11}$$

The frozen coefficients are

$$d_0 = 1.1971212004620583 \times 10^{-11}, \qquad d_1 = 9.87022611239063 \times 10^{-14},$$
$$d_2 = -1.0336424891583054 \times 10^{-13}, \qquad d_3 = 1.189490499106138 \times 10^{-10},$$
$$d_4 = 1.2236365189926054 \times 10^{-10}.$$

Printed digits retain the source point; they do not express experimental precision. H_* fixes this normalization and is not the Hubble rate of the static background below. The source label screened denotes the archived portal parameter choice, not an all-orders radiative-protection result. The generic portal defaults of the archived `Params` class describe a different benchmark and are not used here.

3 Dynamics, constraints and local validity

3.1 Matter and optional gravitational equations

Partial derivatives of $P(X, \sigma, \theta)$ hold the other listed arguments fixed. Variation of (2) gives

$$\Box\sigma + P_\sigma - \tfrac{1}{2}(\partial_\sigma m_\chi^2)\chi^2 = 0, \tag{12}$$
$$\nabla_\mu(P_X \nabla^\mu\theta) + P_\theta - \tfrac{1}{2}(\partial_\theta m_\chi^2)\chi^2 = 0, \tag{13}$$
$$\Box\chi - m_\chi^2\chi = 0. \tag{14}$$

For the selected theory,

$$P_X = K + 2aX, \qquad P_{XX} = 2a, \tag{15}$$
$$P_\sigma = K_{,\sigma}X + a_{,\sigma}X^2 - U_{,\sigma}, \qquad P_\theta = -U_{,\theta}, \tag{16}$$
$$P_{X\sigma} = K_{,\sigma} + 2a_{,\sigma}X, \qquad P_{X\theta} = 0. \tag{17}$$

The stress tensor is

$$T_{\mu\nu} = \nabla_\mu\sigma\nabla_\nu\sigma + P_X\nabla_\mu\theta\nabla_\nu\theta + \nabla_\mu\chi\nabla_\nu\chi + g_{\mu\nu}\mathcal{L}_\mathrm{m}. \tag{18}$$

For a fixed metric these are matter equations. If gravity is dynamical, the additional postulate is minimal Einstein coupling,

$$S_\mathrm{g} = \int \mathrm{d}^4x\sqrt{-g}\,\frac{M_\mathrm{Pl}^2}{2}R, \qquad M_\mathrm{Pl}^2 G_{\mu\nu} = T_{\mu\nu}, \tag{19}$$

with curvature convention $G_{00} = 3H^2$ in flat FRW. This matches the archived FRW convention and does not derive spacetime or Einstein gravity from the scalar fields. On the matter equations, diffeomorphism invariance gives $\nabla_\mu T^{\mu\nu} = 0$.

In flat FRW with homogeneous fields, $X = \dot\theta^2/2$ and

$$\rho = \tfrac{1}{2}\dot\sigma^2 + 2XP_X - P + \tfrac{1}{2}\dot\chi^2 + \tfrac{1}{2}m_\chi^2\chi^2, \tag{20}$$
$$p = \tfrac{1}{2}\dot\sigma^2 + P + \tfrac{1}{2}\dot\chi^2 - \tfrac{1}{2}m_\chi^2\chi^2, \tag{21}$$
$$3M_\mathrm{Pl}^2H^2 = \rho, \qquad -2M_\mathrm{Pl}^2\dot H = \rho + p, \qquad \dot\rho + 3H(\rho + p) = 0. \tag{22}$$

The P-sector energy-density identity is standard in k-essence [6]; the canonical sectors and portal here are retained explicitly. The field equations reduce to

$$\ddot\sigma + 3H\dot\sigma - P_\sigma + \tfrac{1}{2}(\partial_\sigma m_\chi^2)\chi^2 = 0, \tag{23}$$
$$D\ddot\theta + 3HP_X\dot\theta + P_{X\sigma}\dot\sigma\dot\theta - P_\theta + \tfrac{1}{2}(\partial_\theta m_\chi^2)\chi^2 = 0, \tag{24}$$
$$\ddot\chi + 3H\dot\chi + m_\chi^2\chi = 0, \tag{25}$$

where $D = P_X + 2XP_{XX} = K + 6aX$. The mixed term in (24) is required by the σ dependence of the phase kinetic coefficient. The Friedmann equation constrains the initial data; it is not an independent freely assignable initial Hubble rate. These classical homogeneous equations are not the complete operational quantum map, in which renormalized χ mode expectations supply separate source terms.

3.2 Domain and initial data

The adopted scope is a local classical theory at tree level, about a specified smooth background. The scale Λ labels an operator coefficient. It does not establish a sharp physical cutoff, a bound on every omitted operator, or a controlled quantum expansion. A quantum use would require a state, renormalization prescriptions and additional counteroperators.

For timelike or zero $\nabla\theta$, impose

$$X \geq 0, \qquad P_X = K + 2aX > 0, \qquad D = K + 6aX > 0. \tag{26}$$

These give positive phase kinetic and spatial-gradient coefficients in the timelike rest frame. Since $a \geq 0$, $K > 0$ is a sufficient restriction for all such X; it is not necessary at every nonzero X. For nonzero null or spacelike gradients the full principal symbol and a common Cauchy hypersurface must be checked instead. The timelike formula below cannot be extended to that class by assertion.

Near the selected constant vacuum we additionally require $m_\chi^2 > 0$, a positive canonically normalized σ–θ Hessian and sufficiently small fluctuations that remain inside this patch. These local conditions do not prove stability of every interacting or gravitational perturbation. At $m_\chi^2 < 0$, the quadratic χ sector diagnoses a tachyonic onset but provides no bounded nonlinear endpoint. The unrestricted field space is not claimed healthy: K can also change sign at sufficiently negative z.

An initial-value problem must specify the fields and normal derivatives on a common spacelike hypersurface, compatible spatial boundary conditions, and, when the metric is dynamical, geometry and extrinsic data satisfying the gravitational constraints. Equations (2)–(11) do not select a unique cosmological trajectory or quantum state. No new historical cosmological evolution is computed here.

4 Consequences of the selected action

4.1 Characteristic cone and energy condition

The characteristic construction is standard for noncanonical scalars [7]. Linearizing the highest-derivative part of (13) gives

$$\mathcal{G}_\theta^{\mu\nu} = P_X g^{\mu\nu} - P_{XX}\nabla^\mu\theta\nabla^\nu\theta. \tag{27}$$

The canonical σ and χ equations have the metric principal cone. Their couplings to θ introduce lower-derivative mixing but no off-diagonal second-derivative terms in this formulation.

Proposition 1 (Local cone and a flat-FRW restriction)**.** *On a timelike-gradient background satisfying (26), the phase characteristic speed in its local rest frame is*

$$c_\theta^2 = \frac{P_X}{D} = \frac{K + 2aX}{K + 6aX}, \qquad 0 < c_\theta^2 \leq 1. \tag{28}$$

At zero gradient the same expression gives the metric cone provided $K > 0$. Throughout a region with $P_X > 0$, the matter stress satisfies the metric null energy condition. Consequently, a spatially flat FRW solution with the Einstein coupling (19) cannot undergo a nonsingular bounce requiring $\dot{H} > 0$ at $H = 0$ while remaining in that domain.

Proof. In the timelike rest frame, (27) has coefficients $(-D, P_X, P_X, P_X)$. Its characteristic equation therefore yields (28). Positivity follows from (26); the upper bound follows from $D - P_X = 4aX \geq 0$. Equality occurs at $aX = 0$. At zero gradient the symbol is $Kg^{\mu\nu}$. For any metric-null vector n^μ, (18) gives

$$T_{\mu\nu}n^\mu n^\nu = (n\cdot\nabla\sigma)^2 + P_X(n\cdot\nabla\theta)^2 + (n\cdot\nabla\chi)^2 \geq 0. \tag{29}$$

In homogeneous FRW, $\rho + p = \dot{\sigma}^2 + 2XP_X + \dot{\chi}^2 \geq 0$, hence (22) implies $\dot{H} \leq 0$. This contradicts the stated bounce condition inside that domain. □

This is a characteristic statement about the displayed local equations, not a frequency-independent group velocity for every massive mixed normal mode. It also differs from a low-frequency sound speed after other fields have been integrated out; controlled heavy-mode elimination has separate assumptions [8]. Its physical use needs a frequency range below a justified EFT cutoff and above masses, relevant mixing rates and background-variation scales. No such nonempty physical window is established by the formal principal symbol alone. It does not derive the measured value of c, a shared cone for unspecified matter, global causal completeness, or the exact ultraviolet front velocity of a completed theory. The bounce restriction assumes spatial flatness, minimal Einstein coupling and the healthy domain; it does not exclude constructions that change those assumptions.

4.2 Constant matter background and retained spectrum

At $X = \chi = 0$, the stationary matter equations are

$$z_0^3 - z_0 + b_\theta(\theta_0) = 0, \qquad E_c z_0 \frac{b_1}{M_{\rm Pl}} + V_s'(\theta_0) = 0. \tag{30}$$

The existing negative-z source solution is

$$\sigma_0/M_{\rm Pl} \simeq -0.02886751345948129, \tag{31}$$

$$\theta_0/M_{\rm Pl} \simeq -0.7301292774006391, \qquad z_0 \simeq -1.1547005383792515. \tag{32}$$

This is distinct from the positive-z cosmological initial branch. It supplies no trajectory connecting the two.

With $q = (\delta\sigma, \delta\theta)^T$, define

$$\mathcal{K}_0 = \mathrm{diag}(1, K_0), \quad K_0 = 1 + \frac{5E_c z_0}{\Lambda^4} \simeq 0.9009694329240460, \tag{33}$$

and the potential Hessian

$$\mathcal{H}_0 = \begin{pmatrix} E_c(3z_0^2 - 1)/f_\sigma^2 & E_c b_1/(f_\sigma M_{\rm Pl}) \\ E_c b_1/(f_\sigma M_{\rm Pl}) & V_s''(\theta_0) \end{pmatrix}. \tag{34}$$

The quadratic local matter Lagrangian is

$$\mathcal{L}_{\rm m}^{(2)} = -\tfrac{1}{2}\nabla q^T \mathcal{K}_0 \nabla q - \tfrac{1}{2} q^T \mathcal{H}_0 q - \tfrac{1}{2}(\nabla\delta\chi)^2 - \tfrac{1}{2} m_{\chi,\rm vac}^2 (\delta\chi)^2. \tag{35}$$

The eigenvalues of $\mathcal{K}_0^{-1/2}\mathcal{H}_0\mathcal{K}_0^{-1/2}$ are the two canonically normalized squared masses. Existing S06 calculations and their independent checks [4] give

$$\frac{m_1^2}{M_{\rm Pl}^2} \simeq 2.90211845795\times10^{-10}, \quad \frac{m_2^2}{M_{\rm Pl}^2} \simeq 2.30400046423\times10^{-9}, \quad \frac{m_{\chi,\rm vac}^2}{M_{\rm Pl}^2} \simeq 9.57111\times10^{-9}. \tag{36}$$

These numbers are reused from the checked record, not presented as newly computed trajectories or experimental masses. Positive curvature does not imply an asymptotically stable particle: the original interaction analysis permits $\phi_2 \to \phi_1\phi_1$. The local fixed-metric expansion is also not an assertion of an exact Minkowski solution of dynamical gravity. The value of $U(\sigma_0, \theta_0)$ must enter Einstein's equations; near-zero cancellation with printed coefficients is not an exact vacuum-energy identity.

The earlier preprint/specification statement [1] $P_X(0) = 1$ is corrected to $P_X(0) = K_0$. Nevertheless, $D = K_0$ at $X = 0$, so $c_\theta^2 = 1$ survives. Correcting normalization is essential for the mass matrix and interactions even when that ratio is unchanged.

5 A concrete conditional test perspective

A test must bind an observable to the fields and background of this action. Consider a region with a calibrated timelike phase gradient and a resolved local rest frame. Independent inputs would be the background z, X, the parameters entering K, a, the reference metric propagation speed, a detector coupling that selects the phase perturbation, and a frequency window in which the principal approximation and EFT truncation both apply. Equation (28) then predicts its characteristic cone. A measured characteristic inconsistent with that relation, after jointly controlled calibration and model errors, would reject this fixed model in that specified regime.

For the sufficient subdomain $K > 0$, let $q_X = aX/K \geq 0$. The same relation reads

$$c_\theta^2 = \frac{1 + 2q_X}{1 + 6q_X}, \qquad q_X = \frac{1 - c_\theta^2}{6c_\theta^2 - 2} \qquad (\tfrac{1}{3} < c_\theta^2 \leq 1). \tag{37}$$

The inverse supplies only the combination q_X; a speed measurement alone cannot determine all action parameters or the background. The lower limiting value is approached as q_X grows and is not a license to take arbitrarily large X outside a controlled EFT regime. This additional range uses $K > 0$, not just the more general conditions (26).

No detector coupling, accessible experimental system or justified measurement-error budget is derived here. Thus this is a specified falsification relation and an input list for a future test, not an immediately executable laboratory proposal or empirical validation. A cosmological application would additionally need constrained initial data, a quantum state where relevant, transfer functions and controlled backreaction/omitted-operator errors. The benchmark in (36) gives another conditional forward target once a physical identification is supplied; assigning its modes to observed particles is presently unsupported.

6 Relationship to the existing inverse-theory study

The separate P01 study, *Identifiability and calibration limits of a single dispersion branch in a two-field effective theory*, [9] provides a useful methodological comparison: specifying a forward model, identifying its parameters from exact observations, and obtaining a reliable inference with finite errors are different tasks. That study uses its own fixed-metric action and rolling background. Its exact reconstruction results and classical matching example therefore cannot be imported into the action of this paper without an explicit map of fields, coefficients, background and observation protocol.

Its retrospective plasmon-polariton application likewise does not validate Apeiron. In that application the unobserved upper gap is not independently certified, calibration freedom remains relevant, and deterministic compatibility thresholds are not experimental confidence intervals. We retain the existing P01 publication and reproduction record as supporting material, without recalculating its historical fits or reinterpreting its dataset as a test of the present core. The local companion's reproduction index gives the exact source paths and checksums.

7 Limits and disposition of broader claims

The mathematical core consists of (2)–(11), the conventions and local domain. Its equations, characteristic relation, conditional energy-condition consequence and canonical mass recipe follow from that same core. The frozen numerical point is an input choice with previously checked outputs. This separation prevents a postulate from being presented as a discovery about nature.

Several historical variants are excluded from this version: a local reconstructed phase potential in place of the global quartic, unscreened generic portal defaults, and an optional stabilizing χ^4 interaction. Adding any of them changes the specified theory or benchmark and requires a

new explicit analysis. The original positive-z cosmological branch and negative-z local vacuum also carry different background assumptions.

The optional additional heavy scalar with source $J = g_0(z-z_c)X$ is a conditional completion ansatz. Saturating the displayed aX^2 coefficient fixes $g_0^2/(2M_S^2) = 5/(3\Lambda^4)$, not an absolute mediator mass. Such an exchange must replace the saturated operator instead of double-counting it. A direct four-field contact coefficient is not the complete massive coupled-channel scattering amplitude, and a massless half-bound diagnostic is not an established sharp EFT cutoff. General analyticity/positivity arguments have their own scattering assumptions and do not make the sign of one coefficient a sufficient UV-completion certificate [10]. A constant Gaussian determinant for a pure heavy-field integral also does not establish vanishing full mixed heavy/light one-loop matching. None of these additional claims is needed for Proposition 1.

The single-carrier interpretation remains a hypothesis. A globally bounded theory, nonlinear reheating endpoint, emergent gravity, a universal causal cone for observed matter, Standard Model or dark-matter identification, black-hole reconversion and a complete quantum response remain open. The preprint also contains a separate post-freeze reconversion extension with an organization scalar and constitutive relaxation equations, but no derived coupling to σ, θ, χ; it remains an explicitly separate, unclosed extension. Conditional entanglement discussion in the latest preprint supplies no missing detector map or new interaction proof [1]. Exact χ parity alone is not evidence for observed dark matter. Existing broader source gates and uncomputed response channels remain unchanged; their incompleteness must not be confused with an inconsistency in the narrower statements actually established here.

The paper claims neither priority for the general k-essence framework nor an empirical confirmation of Apeiron. Its contribution is an explicit source-bound formulation, visible correction of inconsistent normalization, and compatible deductions with stated assumptions. External scientific review, phenomenological adequacy and public release are separate matters.

8 Conclusion

The selected original Apeiron scalar action admits a precise local classical formulation without replacing its potential or portal. Retaining the derivative-dependent tilt fixes its phase normalization. The same action yields a conditional subluminal phase characteristic and a null-energy restriction on healthy spatially flat Einstein-FRW evolution. The existing frozen spectrum is a reproducible local benchmark. These results make the limited theory assessable while exposing the additional assumptions required for its broader physical interpretation.

A Explicit derivatives and variational checks

For direct implementation of the field equations,

$$K_{,\sigma} = \frac{5E_c}{f_\sigma \Lambda^4}, \qquad a_{,\sigma} = \frac{10(z-z_c)}{3f_\sigma \Lambda^4}, \tag{38}$$

$$U_{,\sigma} = \frac{E_c}{f_\sigma}(z^3 - z + b_\theta), \qquad U_{,\theta} = \frac{E_c b_1 z}{M_{\rm Pl}} + V_s'(\theta), \tag{39}$$

$$(\partial_\sigma m_\chi^2) = 4\lambda(v+\sigma)\cos\left(\frac{2\theta}{f_{\rm phase}}\right), \qquad (\partial_\theta m_\chi^2) = -\frac{4\lambda(v+\sigma)^2}{f_{\rm phase}}\sin\left(\frac{2\theta}{f_{\rm phase}}\right). \tag{40}$$

Writing $\delta X = -\nabla^\mu\theta\nabla_\mu\delta\theta$ in the action and integrating once by parts yields (13). Expanding its first term gives

$$\nabla_\mu(P_X\nabla^\mu\theta) = \mathcal{G}_\theta^{\mu\nu}\nabla_\mu\nabla_\nu\theta + P_{X\sigma}\nabla_\mu\sigma\nabla^\mu\theta + P_{X\theta}\nabla_\mu\theta\nabla^\mu\theta. \tag{41}$$

The final term vanishes for this model. Evaluating the middle term on a homogeneous background supplies the mixed term in (24) after multiplication by minus one. Metric variation supplies (18); contraction with a null vector removes its $g_{\mu\nu}\mathcal{L}_{\mathrm{m}}$ term and yields (29).

B Evidence and reproduction scope

The accompanying English claim/source matrix binds each main claim to the exact internal source and its role: postulate, derivation, reused numerical output or open extension. The reproduction index separates rebuilding this PDF from reproducing historical scientific calculations. Historical original files and frozen outputs are unchanged. Their references and hashes identify the prior spectrum and normalization checks. No historical trajectory, full spectrum or loop calculation was rerun for this consolidation. The bounded convention check adds 14 exact rational-polynomial identities and one explicitly labelled floating-point benchmark illustration, which is not a new vacuum search or spectrum computation.

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